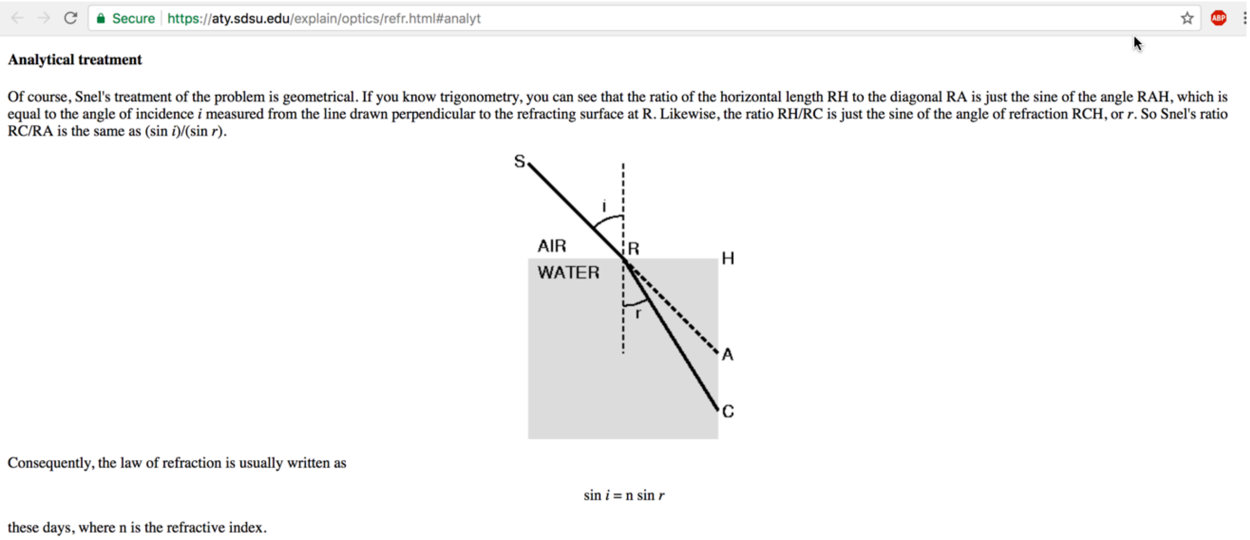
Calculation of Astronomical Refraction

Introduction

To calculate the refraction, we must know the refractive index in the region through which the rays of light pass. For astronomical refraction, this is the whole atmosphere; so some simplification is helpful. Most calculations assume that the atmosphere is spherical, and that the surfaces of constant density are concentric spheres.



Analytical treatment

Of course, Snel's treatment of the problem is geometrical. If you know trigonometry, you can see that the ratio of the horizontal length RH to the diagonal RA is just the sine of the angle RAH, which is equal to the angle of incidence *i* measured from the line drawn perpendicular to the refracting surface at R. Likewise, the ratio RH/RC is just the sine of the angle of refraction RCH, or *r*. So Snel's ratio RC/RA is the same as (sin *i*)/(sin *r*).

Consequently, the law of refraction is usually written as

$$\sin i = n \sin r$$

these days, where n is the refractive index.

The atmosphere isn't really spherical, not only because the Earth is an oblate spheroid instead of a sphere, but also because the atmosphere is dynamic and contains lateral gradients of temperature and pressure. However, the deviations from sphericity are small, and can be neglected for many purposes.

A spherically-stratified atmosphere greatly simplifies the work, because instead of three spatial coordinates, we need only deal with one (the distance from the center). Furthermore, the spherical model greatly simplifies the mathematical problem, because it has a quantity that remains constant along the entire length of a refracted ray. This refractive invariant is so important that I have devoted a separate page to it.

Procedure

Here's an outline of what's involved in doing the refraction calculations:

Model atmosphere

First, we have to specify the atmospheric structure completely. As discussed on another page, this can be done if we specify the surface pressure, and the temperature everywhere as a function of height.

There are physical constraints on thermal profiles: not everything one can imagine is physically possible, or even plausible. These thermal considerations are discussed on another page.

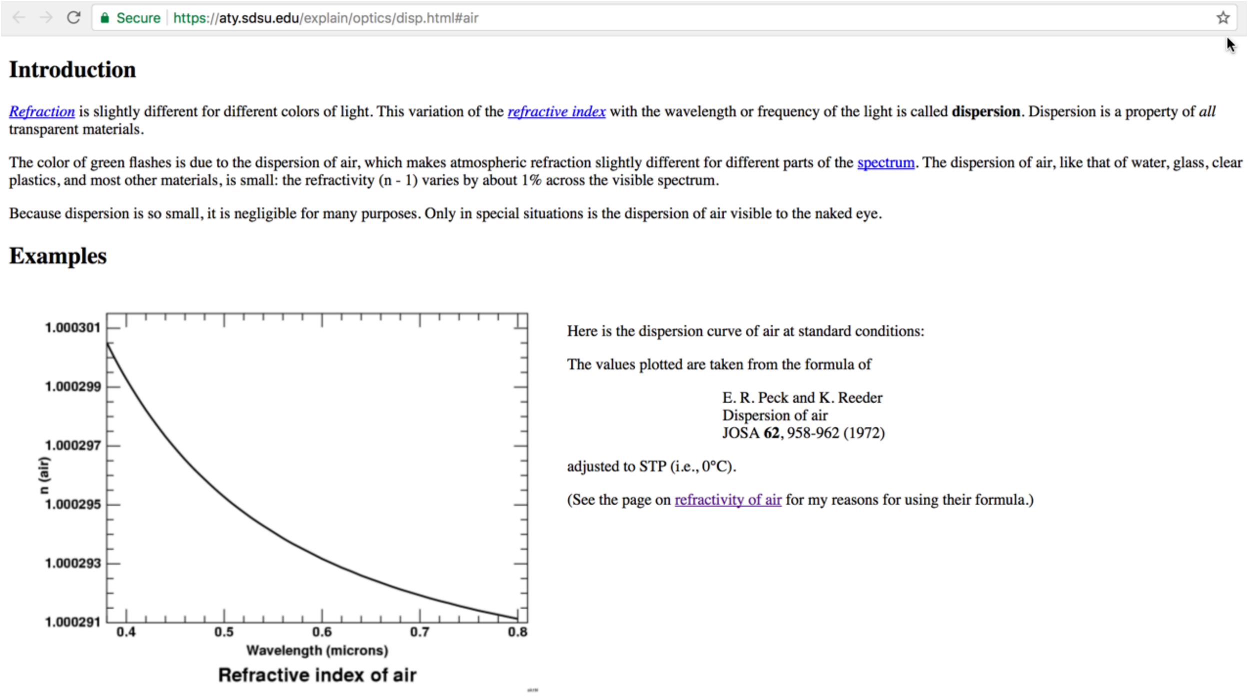
Density profile

Given the surface pressure and the temperature profile, calculate the density as a function of height. The density is found by combining the perfect gas law with the principle of hydrostatic equilibrium(which is just the assumption that the pressure at every point is due to the weight of the overlying gas).

Of course, the gas law requires a mean molecular weight, which means we have to specify the composition of the gas. But we have to specify this anyway, to calculate the refractive index from the density. The effects of humidity are generally so small that most calculations are done for dry air of standard composition; the carbon dioxide content also has a small effect.

Refractivity profile

Once the density and composition are specified everywhere, the refractivity (n − 1) can be calculated for a given wavelength of light. The dispersion curve for standard dry air at STP is very well knownfrom extremely accurate laboratory measurements. This is scaled by the density — an assumption that is usually referred to as the "Gladstone-Dale Law", though it is not in accord with theoretical calculations, and (even more strangely) Gladstone and Dale actually investigated liquids and not gases. However, it appears to be a satisfactory approximation, and is certainly good enough for our work here.



Introduction

Refraction is slightly different for different colors of light. This variation of the *refractive index* with the wavelength or frequency of the light is called **dispersion**. Dispersion is a property of *all* transparent materials.

The color of green flashes is due to the dispersion of air, which makes atmospheric refraction slightly different for different parts of the spectrum. The dispersion of air, like that of water, glass, clear plastics, and most other materials, is small: the refractivity (n - 1) varies by about 1% across the visible spectrum.

Because dispersion is so small, it is negligible for many purposes. Only in special situations is the dispersion of air visible to the naked eye.

Examples

Refractive index of air

Here is the dispersion curve of air at standard conditions:

The values plotted are taken from the formula of

E. R. Peck and K. Reeder
Dispersion of air
JOSA **62**, 958-962 (1972)

adjusted to STP (i.e., 0°C).

(See the page on refractivity of air for my reasons for using their formula.)

The refractivity depends on wavelength in a well-known way. This dispersion of the refractivity is the basic cause of green flashes. The refractivity varies by about 1% from red to green light; but even this small difference is enough to produce spectacular results, given the proper atmospheric structure.

Note that it is the *refractivity*, rather than the refractive *index*, that is proportional to the density. The refractive index is 1 + refractivity; and, as the latter is usually less than 0.0003, the refractive index varies by about 1 part per million for each Celsius degree of temperature change. (Even so, it turns out that very small thermal structures can have appreciable effects if they produce locally steep temperature gradients; so millidegrees, and even smaller differences, need to be retained in the calculations.)

Calculate a refraction table

Given the refractivity profile, we can calculate the refraction at a given apparent altitude in the sky. In principle, this involves solving a differential equation; in practice, it is reduced to evaluating a definite integral numerically. The basic technique is described in a paper by **Auer and Standish**, which is available in a Web archive. There are a number of pitfalls, involving cancellation of leading significant figures, and the consequent magnification of roundoff errors, that will trap the unwary at this stage. These details are too technical to go into on a Web page, but are critical in obtaining useful results.

For reducing astronomical observations of position, the refraction table is what is needed. However, to understand green flashes and other low-Sun distortions, it is necessary to construct images as well.

Construct images

These additional steps are needed to interpret sunset phenomena:

Convert the refraction table to a transfer curve

Because we can only calculate refraction as a function of *apparent* altitude, the “true” or geometric altitude that is seen at a given apparent altitude must be found by subtracting the refraction from the apparent altitude. The relation between true and apparent altitude (i.e., the transfer curve) then allows us to determine what parts of the Sun appear in what parts of the sky.

Convert the transfer curve to images

By finding what part of the Sun's disk is seen at a given apparent altitude, we can construct an image of the distorted (and possibly miraged) Sun. The method of doing this is clearly explained byWegener in his **1918 paper**.

The result is, of course, a monochromatic image of the Sun. We need several such images to construct a color picture.

Combine monochromatic images into a colored image

I have described an approximate way to do this on another page.

Combine many colored images to make an animation

I have also described how to do this on that other page.

Improvements

Although the simulations are interesting and useful, they are not accurate in many respects: the colors are only approximate, and many details that influence the actual appearance of the low Sun (such as limb darkening and atmospheric reddening and extinction) have been omitted.

It's useful to incorporate these additional phenomena. They are complicated; however, the correlation of refraction with *airmass* and reddening (in the form of Laplace's extinction theorem) means that photographs contain additional information. Some initial attempts to produce such realistic images are now appearing among my sunset simulations. Unfortunately, I'm not yet able to animate them.